



Modeling the Transmission Dynamics of Maize Foliar Disease in Maize Plants

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Abstract

Maize Foliar Disease (MFD) is a major global problem for maize crop production, causing significant losses in both yield and quality. This study formulates and analyzes a mathematical model to understand MFD transmission dynamics in maize plants. The disease-free equilibrium state of the model was determined, and the basic reproduction number (\mathcal{R}_0) for MFD was computed using the next-generation matrix method. The analysis reveals that \mathcal{R}_0 is sensitive to infection rates of susceptible maize plants and susceptible pathogens, establishing that the disease-free state is locally asymptotically stable when \mathcal{R}_0 is less than 1 but unstable when \mathcal{R}_0 exceeds 1. The findings indicate that increasing the natural death rate of infected pathogens can significantly decrease MFD spread. Effective strategies, such as implementing disease-resistant maize varieties and enhancing crop management practices, play a crucial role in lowering infection rates, highlighting the importance of targeted agricultural practices in mitigating the impact of MFD and enhancing maize production.

Keywords: Maize Foliar Disease, Maize plants, Transmission dynamics, Mathematical modelling.
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1. Introduction

Maize (*Zea mays L.*) is a highly valuable crop globally, with its kernels serving as a crucial food source for both humans and animals [1]. It serves as a fundamental dietary staple in numerous regions across the globe [2]. Maize cultivation encompasses both temperate and tropical regions on every continent around the globe [3]. Maize stands as the foremost staple cereal globally, with annual production surpassing 1 billion metric tons [11]. Maize is cultivated annually across approximately 15.5 million hectares in Africa, predominantly by small-scale farmers who rely on it for both their livelihood and food security [5]. Maize occupies a central position as the predominant cereal crop in

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Tanzania, accounting for a significant 70% of the overall cereal production [9]. In more affluent areas where commercial cultivation is practiced, maize is typically planted using mechanical methods, whereas in many other parts of the world, manual planting with a hoe remains common practice [56].

Maize yield losses can be attributed to a variety of factors such as abiotic stresses (drought, low soil fertility, high plant density) and biotic stresses [12]. Biotic stresses such as pests and diseases, negatively impact both the quality and quantity of maize [58]. Maize plants can be affected by a variety of diseases at any stage of their life cycle [59, 29]. Corn diseases can be categorized based on their types into fungal, bacterial, parasitic, nematode, viral, or virus-related diseases. Foliar diseases are one of the most damaging fungal diseases to maize production globally [13]. They can reduce the quality and quantity of grain produced by the plant, and are commonly caused by higher order fungi [14]. Common foliar diseases of maize are: northern leaf blight, southern rust, southern leaf blight, gray leaf spot, eyespot, common rust, head smut and *Diplodia* leaf streak [15]. These diseases are responsible for losses in total yield and reduce the efficacy of the harvest [16]. In instances of severe infection, foliar diseases can result in a purchase of silage bags or require the entire plant to be harvested for silage [20]. All of this can result in increased expenses for the farmer and a reduction in the quality of the feed.

Understanding and combatting foliar diseases in maize crops requires a multifaceted approach [51, 52]. Utilizing resistant maize varieties and implementing control measures against disease-carrying vectors, such as leafhoppers, through insecticides or integrated pest management strategies are essential steps in managing these diseases [21]. Early detection methods along with the prompt eradication of infected plants and strict quarantine measures supplemented by cultural practices like adequate spacing, play crucial roles in preventing the spread of foliar diseases within maize fields [22].

Despite concerted efforts, foliar diseases persist as a notable challenge, affecting maize cultivation in areas such as Tanzania and Sub-Saharan Africa [60]. Researchers have turned to mathematical modeling as a valuable approach for understanding the dynamics of infectious diseases including foliar diseases in maize [23]. Different mathematical models have been developed to explore into the intricate changes of diseases as time progresses, aiding in the development of targeted strategies for disease management and prevention [19, 18, 17, 4, 43, 7].

For instance, Ali et al. [64] explored the effects of curative and preventive control measures in plant disease models. Their study found, control measures, curative and preventive offer an innovative point of view on the behavior dynamics of infectious diseases in plants. Nisar et al. [65] reviewed fractional order epidemic models in life sciences, emphasizing their dynamic impact on disease analysis, decision-making, and control. Anggriani et al. [63] developed a mathematical model showing that using curative and preventive treatments simultaneously can reduce plant disease infections. Holt et al. [55] created a mathematical model to analyze the spread of disease caused by the African mosaic virus. The study suggested that using infected cutting tools and removing infectious cassava have limited impact on the disease occurrence. Meng and Li [57] found impulsive removal of diseased plants more effective and cost-efficient than continuous removal in plant disease dynamics.

Collins et al. [24] investigated optimal control strategies for foliar disease dynamics

across multiple maize varieties, highlighting that early intervention at the onset of an outbreak is most effective and cost-efficient in reducing disease spread. The study developed by Kumar et al. [25] on a novel mechanism to simulate fractional order maize foliar disease dynamical model founded that the control methods are more applicable and more effective in controlling foliar disease. Putri et al. [26] showed that a maize foliar disease model with a standard incidence rate is determined by its basic reproductive ratio.

Kapange et al. [27] analyzed the transmission dynamics of northern corn leaf blight disease, highlighting the influence of seasonal weather variations on pathogen transmission, shedding rate, and maize plant mortality due to the disease. Alemneh et al. [28] developed an eco-epidemiological model for analyzing MSV disease transmission dynamics in maize, demonstrating that control interventions effectively reduce disease infections. Melese et al. [6] studied mathematical modeling of coffee berry disease dynamics on farms, suggesting that understanding disease transmission and eradication dynamics requires assuming logistic growth with a carrying capacity reflecting the farm's size limitation.

Taking inspiration from the previously mentioned studies, we have formulated a mathematical model to delve into the transmission dynamics of foliar diseases in maize plants. Notably, our investigation aims to develop strategies that can help manage the transmission of these diseases. Specifically, we explore the impact of various factors on the dynamics of foliar disease transmission. To our knowledge, this study represents a significant attempt to comprehensively analyze foliar disease transmission dynamics in maize plants, thus contributing novel insights to the field.

2. Maize foliar disease model formulation

This study aims to develop a mathematical model for analyzing maize foliar disease dynamics and proposing cost-effective control strategies. Plant viruses, as noted by Jeger et al. [61], spread through host-vector systems. Building on Collins and Duffy's model [24], the proposed model divides the system into maize plants and pathogens. Maize plants are categorized into susceptible $S(t)$ and infected $I(t)$ classes, while foliar disease pathogens are categorized into susceptible $X(t)$ and infected $Y(t)$ classes. The model is constructed based on these assumptions:

- The mortality rate of the maize population remains constant over time.
- The planting rate of susceptible maize populations remains constant.
- Infected maize plants die at a constant rate due to pathogenic infections.
- The increase of the pathogen population is proportional to the number of infected maize plants.
- The infection of susceptible maize by the pathogen remains constant.
- Within the pathogen population, mortality rates remains constant.

2.1. Compartmental Flow Diagram for the MFD Dynamics

This diagram visually depicts disease progression through susceptible and infected plant compartments, aiding in understanding disease transmission and control strategies. Figure 1 shows interactions between susceptible and infected individuals. The parameters indicated in Figure 1 are described in Table 1.

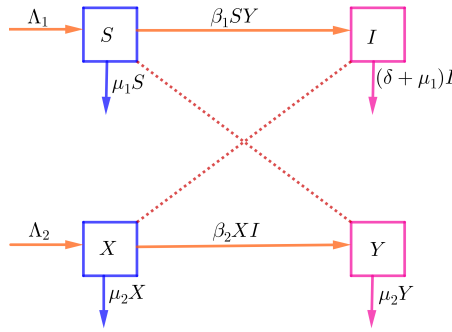


Figure 1: Model flow chart for the dynamics of Maize Foliar Disease

Table 1: Model parameters and their description

Parameter	Description	Value (day ⁻¹)	Source
Λ_1	Maize plants recruitment rate	10	[26]
Λ_2	Pathogens recruitment rate	0.0143	[29]
μ_1	Natural death rate of maize plants	0.0103	[26]
μ_2	Natural death rate of pathogens	0.1236	[29]
β_1	Infection rate of susceptible maize by pathogen	0.0024	[62]
β_2	Infection rate of susceptible pathogen by maize	0.0078	Assumed
δ	MFD mortality rate from infected state	0.0206	[25]

2.2. Model Equations for the MFD Dynamics

The model equations consist of differential equations that describe the rates of change in the populations of susceptible and infected plants over time. These equations incorporate factors such as transmission rates, recovery rates, and control measures to predict the disease spread and evaluate intervention strategies [41]. The model is governed by the

following system of differential equations:

$$\begin{aligned} \frac{dS}{dt} &= \Lambda_1 - (\beta_1 Y + \mu_1)S \\ \frac{dI}{dt} &= \beta_1 SY - (\delta + \mu_1)I \\ \frac{dX}{dt} &= \Lambda_2 - (\beta_2 I + \mu_2)X \\ \frac{dY}{dt} &= \beta_2 XI - \mu_2 Y \end{aligned} \tag{2.1}$$

where: $S(0) > 0$, $I(0) \geq 0$, $X(0) \geq 0$, and $Y(0) \geq 0$.

3. Model Properties

Here, we qualitatively analyze the Maize Foliar Diseases model system (2.1) with time-dependent control parameters.

3.1. Positivity of solution

To ensure model validity, all solutions from positive initial conditions must remain positive for $t \geq 0$. [50]. This will be shown by the following theorem.

Theorem 3.1. *The solution $(S(t), I(t), X(t), Y(t))$ of system (2.1) remains positive for all $t \geq 0$ within $\Omega = (S, I, X, Y) \in \mathbb{R}_+^4 : S(0) > 0, I(0) > 0, X(0) \geq 0, Y(0) \geq 0$.*

Proof. Recalling the first equation in model system (2.1), then we have;

$$\begin{aligned} \frac{dS}{dt} &= -\mu_1 S \\ \frac{dS}{dt} &\geq -\mu_1 S \\ \frac{dS}{S} &\geq -\mu_1 ds \end{aligned} \tag{3.1}$$

By separating the variables, we have;

$$\int_{S(0)}^{S(t)} \frac{dS}{S} \geq - \int_0^t \mu_1 ds$$

$$\ln S(t) - \ln S(0) \geq -\mu_1 t$$

Upon simplifying further, we get;

$$S(t) \geq S(0)e^{-\mu_1 t} \tag{3.2}$$

Thus, it follows that the solution $S(t)$ remains positive for all t as long as the initial susceptible population $S(0)$ is positive and the exponential term $e^{-\mu_1 t}$ remains non-negative. Conversely, it can be proven that $I(0) \geq 0$, $X(0) \geq 0$, and $Y(0) \geq 0$. This completes the proof, ensuring all solutions remain positive for $t \geq 0$, confirming the model's validity. \square

3.2. Invariant region

In this subsection, we explore model variables and confirm a unique bounded solution exists. From (2.1), focusing on maize plants:

$$\frac{dN_1}{dt} = \frac{dS}{dt} + \frac{dI}{dt} \leq \Lambda_1 - \mu N_1 - \delta I. \tag{3.3}$$

Solving this, we obtain

$$0 \leq N_1(t) \leq \frac{\Lambda_1}{\mu} + N_1(0) \exp^{-\mu_1 t}. \tag{3.4}$$

As $t \rightarrow \infty$, we have $0 < N_1(t) \leq \frac{\Lambda_1}{\mu_1}$.

Also considering the pathogen population, $N_2 = X + Y$ we have;

$$\frac{dN_2}{dt} = \frac{dX}{dt} + \frac{dY}{dt} \leq \Lambda_2 - \mu_2 N_2. \tag{3.5}$$

Solving this, we obtain

$$0 \leq N_2(t) \leq \frac{\Lambda_2}{\mu_2} + N_2(0) \exp^{-\mu_2 t}. \tag{3.6}$$

As $t \rightarrow \infty$, we have $0 < N_2(t) \leq \frac{\Lambda_2}{\mu_2}$.

Hence, the feasible solution set for the MFD model is given by

$$\Omega = \Omega_1 \times \Omega_2 = \left\{ (S, I, X, Y) \in \mathbb{R}_+^4 : N_1 \leq \frac{\Lambda_1}{\mu_1} : N_2 \leq \frac{\Lambda_2}{\mu_2} \right\} \tag{3.7}$$

The model's invariant feasibility in \mathbb{R}_+^4 confirms its well-posed nature for further mathematical analysis [53].

3.3. Disease Free equilibrium point(DFE)

The disease-free equilibrium point E^0 , where $S, I, X,$ and Y are zero, is found by setting the equations of model (2.1) to zero:

$$E^0 = (S^0, I^0, X^0, Y^0) = \left(\frac{\Lambda_1}{\mu_1}, 0, \frac{\Lambda_2}{\mu_2}, 0 \right) \tag{3.8}$$

3.4. The Basic Reproduction Number

The basic reproduction number, \mathcal{R}_0 , represents the average secondary infections from one infected individual in a susceptible population [38, 35]. It is calculated as follows:

$$\dot{X}_i = F_i(x) - V_i(x) \tag{3.9}$$

where F_i is the rate of appearance of new infection in compartment i and V_i is the transfer of infections from one compartment i to another. We employ the standard methodology proposed by Diekmann et al. [35, 37] as used in the study conducted by Abdulwasaa et al.[44] to calculate the basic reproduction number for the model system (2.1) as:

$$FV^{-1} = \left[\frac{\partial F_i(E_0)}{\partial x_j} \right] \left[\frac{\partial V_i(E_0)}{\partial x_j} \right]^{-1} \tag{3.10}$$

Equation 2.1 describes the disease compartments as:

$$\begin{aligned}\frac{dI}{dt} &= \beta_1SY - (\delta + \mu_1)I \\ \frac{dY}{dt} &= \beta_2XI - \mu_2Y\end{aligned}\quad (3.11)$$

Expressing the equations in 3.11 in terms of the system of equations given in 3.9, we have:

$$\dot{X}_i = \begin{bmatrix} \beta_1SY \\ \beta_2XI \end{bmatrix} - \begin{bmatrix} (\delta + \mu_1)I \\ \mu_2Y \end{bmatrix}\quad (3.12)$$

which implies that

$$\mathcal{F}_i = \begin{bmatrix} \beta_1SY \\ \beta_2XI \end{bmatrix}$$

and

$$\mathcal{V}_i = \begin{bmatrix} (\delta + \mu_1)I \\ \mu_2Y \end{bmatrix}$$

Thus,

$$F = \frac{\partial \mathcal{F}_i}{\partial (I, Y)} = \begin{bmatrix} 0 & \frac{\beta_1\Lambda_1}{\mu_1} \\ \frac{\beta_2\Lambda_2}{\mu_2} & 0 \end{bmatrix}$$

and

$$V = \frac{\partial \mathcal{V}_i}{\partial (I, Y)} = \begin{bmatrix} \delta + \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}$$

which implies that

$$V^{-1} = \frac{1}{\mu_2(\delta + \mu_1)} \begin{bmatrix} \mu_2 & 0 \\ 0 & \delta + \mu_1 \end{bmatrix}$$

Following computation, the basic reproduction number was determined to be:

$$\mathcal{R}_0 = \sqrt{\frac{\beta_1\beta_2\Lambda_1\Lambda_2}{\mu_1\mu_2(\delta + \mu_1)^3}} = \mathcal{R}_{0_1} \times \mathcal{R}_{0_2}\quad (3.13)$$

where

- (i) $\mathcal{R}_{0_1} = \sqrt{\frac{\beta_1\Lambda_1}{\mu_1}}$ is the number of infected pathogens caused by a typical infectious maize plants.
- (ii) $\mathcal{R}_{0_2} = \sqrt{\frac{\beta_2\Lambda_2}{\mu_2(\delta + \mu_1)^3}}$ is the number of infected maize plants caused by a typical infectious pathogen.

3.5. Local stability of Maize Foliar Disease-Free Equilibrium

Here, we analyze the disease-free equilibrium’s local stability using the eigenvalue method.

Theorem 3.2. *The disease-free equilibrium of model system (2.1) is stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.*

Proof. It can be proved by showing that all eigenvalues of $J(E_0)$ for the MFD-free model are negative.

The model system (2.1) can be written in matrix form as:

$$J(E_0) = \begin{bmatrix} -\Lambda_1 & 0 & 0 & \frac{-\beta_1\Lambda_1}{\mu_1} \\ 0 & -(\delta + \mu_1) & 0 & \frac{-\beta_1\Lambda_1}{\mu_1} \\ 0 & \frac{-\beta_2\Lambda_2}{\mu_2} & -\Lambda_2 & 0 \\ 0 & \frac{\beta_2\Lambda_2}{\mu_2} & 0 & -\mu_2 \end{bmatrix}$$

The Jacobian matrix $J(E_0)$ has four distinct eigenvalues: $\lambda_1 = -\Lambda_1$, $\lambda_2 = -(\delta + \mu_1)$, $\lambda_3 = -\Lambda_2$, and $\lambda_4 = -\mu_2$, all with negative real parts. Thus, all eigenvalues of $J(E_0)$ are negative, indicating local asymptotic stability of the disease-free equilibrium E_0 when $\mathcal{R}_0 < 1$. □

3.6. Global Stability Analysis of the Disease-Free Equilibrium for Maize Foliar Disease

In this subsection, we analyze the global stability of the disease-free equilibrium (DFE) in model system (2.1) using the Lyapunov approach as per Castillo-Chavez et al. [39], Nyerere et al. [31], and Mgandu et al. [32]. Our focus is on establishing a non-positive Lyapunov function derivative to characterize DFE’s global behaviour.

Theorem 3.3. *The MFD-free equilibrium point for the model system (2.1) is globally asymptotically stable on Ω if $\mathcal{R}_0 < 1$.*

Proof. We use the approach in Tewa et al. [42] to define the explicit Lyapunov function candidate L for model system (2.1) as:

$$L = (S - S^* \ln S) + I + X + Y. \tag{3.14}$$

Thus:

$$\begin{aligned} \frac{dL}{dt} &= \left(1 - \frac{S^*}{S}\right) \frac{dS}{dt} + \frac{dI}{dt} + \frac{dX}{dt} + \frac{dY}{dt} \\ &= \left(1 - \frac{S^*}{S}\right) (\Lambda_1 - (\beta_1 Y + \mu_1)S) + \beta_1 S Y - (\delta + \mu_1)I + \Lambda_2 - \beta_2 X I - \mu_2 X + \beta_2 X I - \mu_2 Y \\ &= \left(1 - \frac{S^*}{S}\right) \Lambda_1 - \mu_1 \left(1 - \frac{S^*}{S}\right) S - \delta I - \mu_1 I + \Lambda_2 - \mu_2 (X + Y) \end{aligned}$$

But, at disease-free equilibrium point $S^* = \frac{\Lambda_1}{\mu_1}$, $X^* = \frac{\Lambda_2}{\mu_2}$ and $I = Y = 0$. It follows that:

$$\frac{dL}{dt} = -\mu_1 S \left(1 - \frac{S^*}{S}\right)^2 \quad (3.15)$$

Therefore, $\frac{dL}{dt} \leq 0$, and using LaSalle's extension to Lyapunov's method [40], it confirms global asymptotic stability of the Maize Foliar Disease-free equilibrium E_0 over Ω , with all solutions' limit sets confined to $\{E_0\}$ where $S = S^*$. \square

3.7. MFD Endemic Equilibrium Point

The model's disease equilibrium is found by setting (2.1) equations to zero:

$$\begin{aligned} \Lambda_1 - (\beta_1 Y^* + \mu_1) S^* &= 0 \\ \beta_1 S^* Y^* - (\delta + \mu_1) I^* &= 0 \\ \Lambda_2 - (\beta_2 I^* + \mu_2) X^* &= 0 \\ \beta_2 X^* I^* - \mu_2 Y^* &= 0 \end{aligned} \quad (3.16)$$

Computing endemic equilibrium point (PE) from eqn (3.16), we get;

$$\begin{cases} S^* = \frac{\Lambda_1}{(\beta_1 Y^* + \mu_1)} \\ I^* = \frac{\beta_1 S^* Y^*}{(\delta + \mu_1)} \\ X^* = \frac{\Lambda_2}{(\beta_2 I^* + \mu_2)} \\ Y^* = \frac{\beta_2 X^* I^*}{\mu_2} \end{cases}$$

Therefore, our proposed model possesses at least one solution [10].

3.8. Global Stability of MFD Endemic Equilibrium Points

The global stability of (E^*) was analyzed using a Lyapunov function, where stability is confirmed if its derivative is negative.

Theorem 3.4. *The MFD has a unique endemic equilibrium point E^* for the model system that is globally asymptotically stable if $R_e > 1$ and unstable otherwise.*

Proof. Following Vargas-De-León [46] and Korobeinikov [33, 34], we employ a Lyapunov function for the model system (2.1). The Lyapunov function V is defined as:

$$V(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^n (\mathbf{x}_i - \mathbf{x}_i^*)^2,$$

where \mathbf{x}_i represents the population of the i -th compartment and \mathbf{x}_i^* denotes the endemic equilibrium point. The model system (2.1) includes the positive definite function:

$$F(S, I, X, Y) = \frac{1}{2} \sum_{i=1}^4 (\mathbf{x}_i - \mathbf{x}_i^*)^2,$$

thus expressing the Lyapunov function of the MFD model system as:

$$V = \frac{1}{2}[(S - S^*) + (I - I^*) + (X - X^*) + (Y - Y^*)]^2.$$

Differentiating the function $L(t)$ with respect to time yields:

$$\frac{dV}{dt} = [(S - S^*) + (I - I^*) + (X - X^*) + (Y - Y^*)] \frac{d}{dt} [S + I + X + Y].$$

$$\frac{dV}{dt} = [S + I + X + Y - (S^* + I^* + X^* + Y^*)] \frac{d}{dt} [S + I + X + Y].$$

However,

$$\frac{d}{dt} (S + I + X + Y) = \Lambda_1 + \Lambda_2 - \delta I - (\mu_1 N_1 + \mu_2 N_2).$$

Furthermore,

$$\begin{aligned} \Lambda_1 - \delta I^* - \mu_1 N_1^* + \Lambda_2 - \mu_2 N_2^* &= 0, \\ \Rightarrow (\Lambda_1 + \Lambda_2) - \delta I^* - (\mu_1 + \mu_2)(S^* + I^* + X^* + Y^*) &= 0, \end{aligned}$$

$$(S^* + I^* + X^* + Y^*) = \frac{(\Lambda_1 + \Lambda_2) - \delta I^*}{(\mu_1 + \mu_2)}.$$

Substituting into $\frac{dV}{dt}$ gives

$$\begin{aligned} \frac{dV}{dt} &= \left[N(t) - \frac{(\Lambda_1 + \Lambda_2) - \delta I^*}{\mu_1 + \mu_2} \right] [(\Lambda_1 + \Lambda_2) - \delta I - (\mu_1 + \mu_2)N(t)] \\ \frac{dV}{dt} &= \left[N(t) - \frac{(\Lambda_1 + \Lambda_2) - \delta I^*}{\mu_1 + \mu_2} \right] \left[-(\mu_1 + \mu_2) \left(N(t) - \frac{(\Lambda_1 + \Lambda_2) - \delta I}{\mu_1 + \mu_2} \right) \right] \\ \frac{dV}{dt} &= -(\mu_1 + \mu_2) \left[N(t) - \frac{\Lambda_1 + \Lambda_2}{\mu_1 + \mu_2} + \frac{\delta I^*}{\mu_1 + \mu_2} \right] \left[N(t) - \frac{\Lambda_1 + \Lambda_2}{\mu_1 + \mu_2} + \frac{\delta I}{\mu_1 + \mu_2} \right] \\ \frac{dV}{dt} &\leq -(\mu_1 + \mu_2) \left[N(t) - \frac{\Lambda_1 + \Lambda_2}{\mu_1 + \mu_2} \right]^2 < 0. \end{aligned}$$

$$\frac{dV}{dt} \leq -(\mu_1 + \mu_2) \left[N(t) - \left(\frac{\Lambda_1 + \Lambda_2}{\mu_1 + \mu_2} \right) \right]^2 < 0.$$

Therefore, $\frac{dV}{dt} < 0$, ensuring global asymptotic stability of the endemic equilibrium point E^* . \square

4. Research Findings and Discussion

4.1. Sensitivity Analysis of the MFD model

Strategies aimed at preventing future outbreaks through necessary control measures to halt epidemics are the focus of epidemiological investigations. It is evident that reducing the value of \mathcal{R}_0 is crucial for stemming the disease's spread. Therefore, studying the effects of parameters on changes in \mathcal{R}_0 , as well as implementing control measures in this

direction, holds significant importance. In sensitivity analysis, we assess model parameters' impact on disease transmission and \mathcal{R}_0 . We use the Chitnis *et al.* [30] method to calculate the sensitivity index for a parameter h in \mathcal{R}_0 as:

$$\gamma_h^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial h} \times \frac{h}{\mathcal{R}_0} \quad (4.1)$$

where $\gamma_h^{\mathcal{R}_0}$ denotes sensitivity index for parameter h in \mathcal{R}_0 . Each index's sign in Table 2 determines whether \mathcal{R}_0 increases (positive) or decreases (negative).

Table 2: Sensitivity Indices

Parameter	Index
Λ_1	+0.5000
Λ_2	+0.5000
β_1	+0.5000
β_2	+0.5000
μ_1	-2.0000
μ_2	-1.0000
δ	-2.0000

4.2. Interpretation of the sensitivity indices

A decrease in \mathcal{R}_0 is expected for parameters μ_1 , μ_2 , and δ as they have negative influences. To mitigate disease spread, it's essential to devise strategies and interventions targeting these parameters. Conversely, with the rise in Λ_1 , Λ_2 , β_1 , and β_2 , which contribute positively to disease propagation, \mathcal{R}_0 also increases, making the disease harder to control. Implementing control measures to curb the effects of Λ_1 , Λ_2 , β_1 , and β_2 is crucial to mitigate disease impact and potentially eradicate it from maize plant populations. A sensitivity analysis plot of several parameters with respect to the basic reproduction number \mathcal{R}_0 is shown in Figure 2.

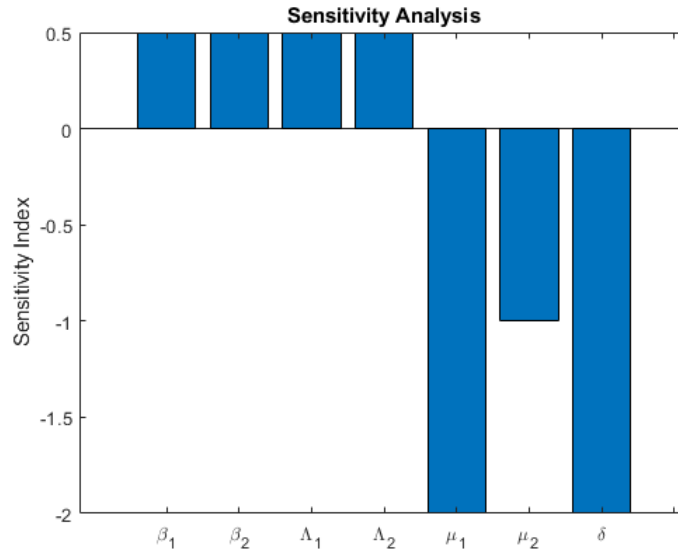


Figure 2: Sensitivity of \mathcal{R}_0 to model parameters.

A positive sensitivity index for parameters like the recruitment rates of maize plants and pathogens, as well as the infection rates, indicates that higher values of these factors lead to a more significant impact on disease transmission. This means that steps to reduce the recruitment and infection rates, such as improving crop management practices to minimize plant stress and implementing disease-resistant maize varieties, can help mitigate the spread of MFD. Conversely, negative sensitivity indices for the natural death rates of maize plants and pathogens, along with the disease mortality rate, suggest that lowering these rates could also contribute to reducing disease transmission. Farmers can focus on strategies to enhance plant health, such as proper irrigation and nutrient management, to decrease plant mortality rates and limit the persistence of pathogens in the environment. By understanding and addressing these influential factors, farmers can implement targeted measures to effectively manage and mitigate Maize Foliar Disease outbreaks in their maize fields, ultimately safeguarding crop yield and quality.

4.3. Numerical Simulation

In this section, MATLAB simulations were conducted to accurately capture MFD transmission dynamics. Initial conditions $S = 100, I = 5, X = 200, Y = 10$ were arbitrarily chosen to influence model behaviour.

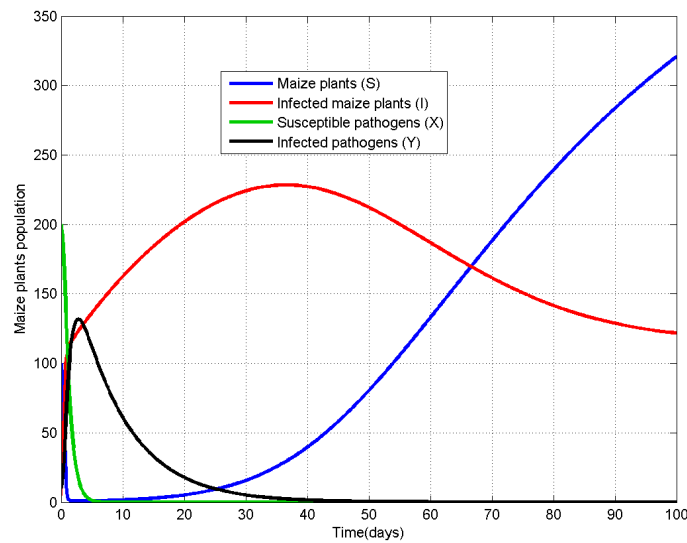


Figure 3: Evolution of Maize Plants with time

In Figure 3, maize plant population initially declines as infection spreads. Subsequent recovery of infected plants and disease control measures lead to gradual population increase, aided by favorable environmental conditions.

As susceptible maize plants encounter infected ones, they become exposed to the virus, reducing susceptibility. Initially low, this population grows as the disease spreads, reaching a plateau where new infections balance with recoveries or mortality. Over time, infected plant numbers may decline due to effective management, natural recovery, or environmental factors.

Likewise, the population of susceptible pathogens is high, especially during periods of favorable environmental conditions and abundant susceptible host plants. However, as time progresses and the disease spreads, susceptible pathogens may encounter several factors that can lead to a gradual decrease in their population. These factors may include natural decay processes, competition with other microbial organisms, or the depletion of susceptible host plants as more plants become infected and recover. Additionally, the implementation of disease management strategies or environmental changes unfavorable to pathogen survival may also contribute to the gradual decrease in the population of susceptible pathogens over time, whereas, the infected pathogens were initially low but as the disease spreads, more pathogens infect plants, leading to an increase in this population over time and due to the reduction in the infected plants the infected pathogens decreases gradually.

4.4. Parameter Variation Simulation Effects on the Model

Here, we vary parameters to explore their impact on system dynamics, focusing on the most sensitive ones.

4.4.1. Impact of Varying Infection Rate β_1 on Maize Foliar Disease Dynamics

Here, we explore how varying β_1 affects Maize Foliar Disease transmission dynamics, shedding light on its role in disease spread among maize plants. Results are depicted in Figure 4.

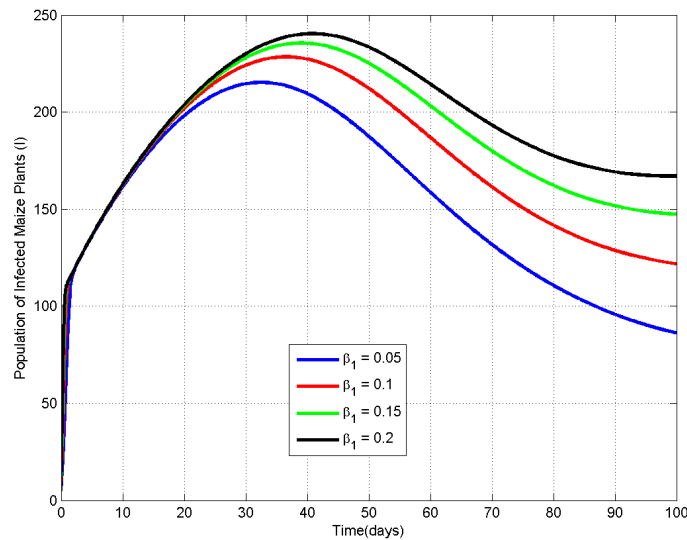


Figure 4: Effects of varying β_1 parameter

Figure 4 shows that as the infection rate β_1 increases, the rate at which susceptible maize plants become infected by the pathogen also increases. This causes a larger proportion of susceptible maize plants to transition to an infected state, leading to a faster spread of the disease within the population. This can reduce the population of susceptible plants as more become infected. Therefore, prioritizing interventions like crop rotation or using resistant maize varieties to lower β_1 is advisable.

4.4.2. Effects of varying natural death rate of pathogens μ_2 on susceptible maize plants

In this simulation, we vary pathogen natural death rate (μ_2) to study its impact on maize plant susceptibility to infection, shedding light on pathogen survival and disease spread dynamics in maize populations. Results are shown in Figure 5.

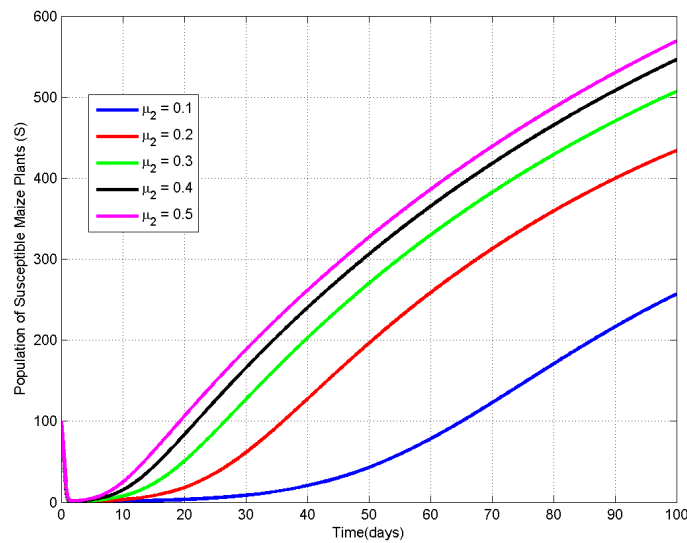


Figure 5: Effects of varying μ_2 parameter

Figure 5 shows that increasing μ_2 reduces susceptible pathogens, lowering disease transmission to maize. Fewer pathogens enhance plant health, potentially boosting crop yields. Higher μ_2 reduces disease outbreaks, improving agricultural outcomes.

4.4.3. Effects of varying natural death rate of maize plants μ_1 on susceptible maize plants

In this simulation, we investigate how adjusting the natural death rate of maize plants (μ_1) influences the vulnerability to becoming infected. The aim is to adjust μ_1 to discern how the lifespan of maize plants affects their likelihood of succumbing to infection by pathogens. This examination provides valuable insights into the relationship between maize plant mortality rates and their vulnerability to Maize Foliar Disease transmission. The findings from this study illuminate the factors shaping disease spread within maize plant populations. The outcomes of this investigation are depicted in figure 6

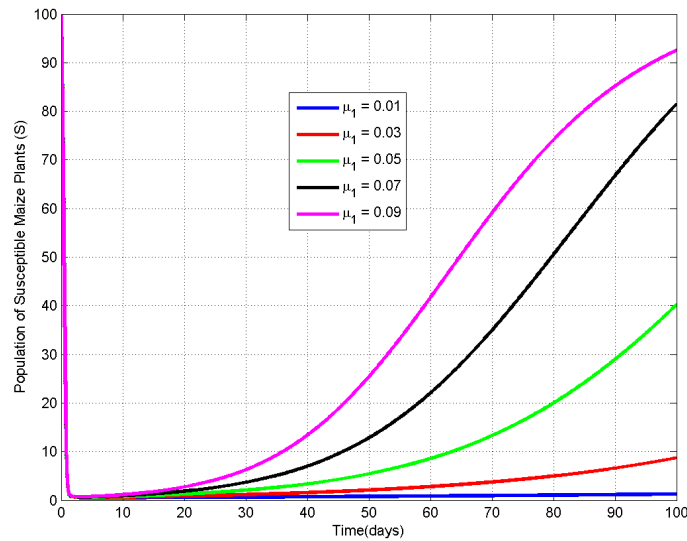


Figure 6: Effects of varying μ_1 parameter

Figure 6 demonstrates how the natural death rate of maize plants, μ_1 , impacts the dynamics of susceptible plants. A higher μ_1 accelerates plant turnover through natural mortality, maintaining a larger susceptible population. This increases the likelihood of Maize Foliar Disease transmission. In contrast, a lower μ_1 slows turnover, reducing the susceptible population and lowering disease transmission risk, thereby enhancing agricultural outcomes.

4.4.4. Combined Impact of Λ_1 and β_1 on \mathcal{R}_0

In this section, we explore how varying the recruitment rate (Λ_1) and infection rate (β_1) of susceptible maize plants collectively influence the basic reproduction number \mathcal{R}_0 of Maize Foliar Disease (MFD). Numerical simulations and graphical analysis demonstrate how different combinations of Λ_1 and β_1 values affect \mathcal{R}_0 , offering insights into factors that promote or inhibit the spread of MFD. Visual representations of these findings can be seen in Figure 7.

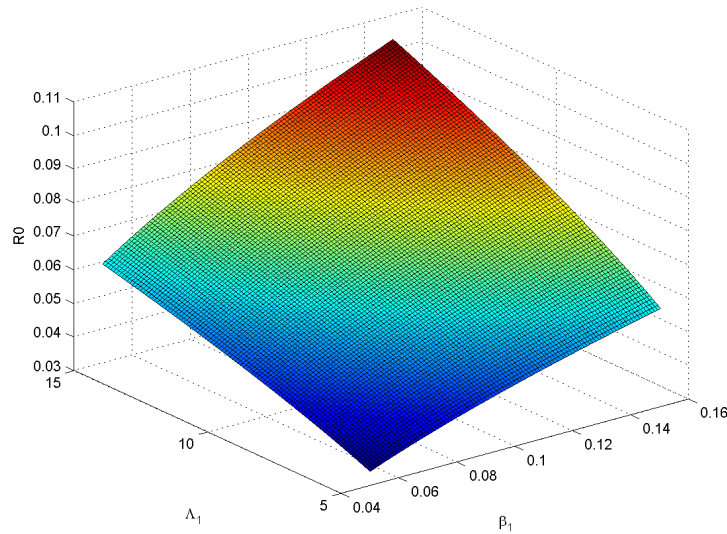


Figure 7: Impact of combined effect of Λ_1 and β_1 on \mathcal{R}_0

In Figure 7, increasing the infection rate (β_1) of susceptible maize plants correlates with higher \mathcal{R}_0 values, indicating greater potential for disease transmission. Conversely, lower β_1 values result in decreased \mathcal{R}_0 , suggesting reduced transmission. Similarly, higher recruitment rates (Λ_1) of maize plants lead to elevated \mathcal{R}_0 , facilitating disease spread, while lower Λ_1 values reduce transmission. The surface plot shows that combinations resulting in higher \mathcal{R}_0 values promote rapid disease spread, whereas lower combinations indicate less favorable conditions. Understanding these dynamics informs agricultural practices to mitigate Maize Foliar Disease, such as optimizing irrigation, maintaining soil health, and employing integrated pest management strategies.

4.4.5. Impact of combined Effect of β_1 and β_2 on \mathcal{R}_0

Here, we aim to examine the combined influence of two pivotal parameters, the effective transmission rate of maize plants (β_1) and the effective transmission rate of pathogens (β_2), on the basic reproduction number (\mathcal{R}_0) of Maize Foliar Disease (MFD). By manipulating the effective transmission rate of maize plants (β_1) and the effective transmission rates of pathogens (β_2), we seek to grasp how alterations in both parameters collectively impact the potential for disease transmission within maize plant populations. Through numerical simulations and graphical analysis, we will elucidate how various combinations of β_1 and β_2 values influence the magnitude of \mathcal{R}_0 , offering insights into the conditions that either facilitate or impede the spread of MFD. The graphical representation in Figure 8 provides a visual depiction of the outcomes, shedding light on the interplay between β_1 and β_2 on \mathcal{R}_0 .

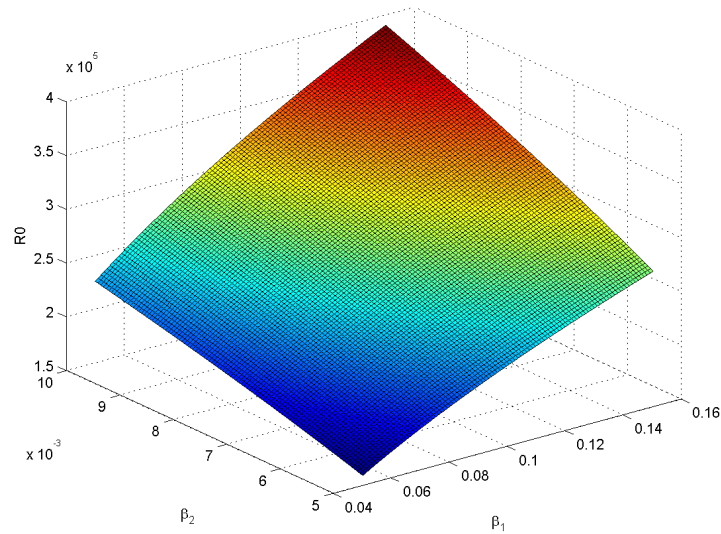


Figure 8: Impact of combined effect of β_1 , β_2 on \mathcal{R}_0

The graph in Figure 8 illustrates the impact of the combined effect of the effective transmission rate of maize plants (β_1) and the effective transmission rate of pathogens (β_2) on the basic reproduction number (\mathcal{R}_0) of Maize Foliar Disease (MFD). As we observe changes in both β_1 and β_2 , the values of \mathcal{R}_0 are depicted across the surface plot. When the effective transmission rate of pathogens (β_2) increases, representing a faster spread of the disease among the plant population, it tends to have a decreasing effect on \mathcal{R}_0 . This suggests that higher rates of pathogen transmission lead to lower values of \mathcal{R}_0 , indicating reduced disease spread within the maize plant population. Conversely, lower values of β_2 result in higher \mathcal{R}_0 values, implying a higher potential for disease transmission. Similarly, variations in the effective transmission rate of maize plants (β_1) also influence the values of \mathcal{R}_0 . Higher values of β_1 correspond to higher \mathcal{R}_0 values, indicating that a faster spread of the disease from infected maize plants contributes to increased disease transmission. Conversely, lower values of β_1 lead to lower \mathcal{R}_0 values, suggesting reduced disease transmission. The surface plot demonstrates how both β_1 and β_2 influence \mathcal{R}_0 together. Higher areas indicate combinations resulting in greater \mathcal{R}_0 , facilitating faster disease spread, while lower areas indicate combinations leading to lower \mathcal{R}_0 , implying reduced disease transmission potential. Understanding these dynamics can offer valuable insights for agricultural strategies. For instance, strategies that target reducing the natural death rate of pathogens or minimizing disease-induced mortality could help in controlling the spread of Maize Foliar Disease, ultimately contributing to better crop yield and quality.

5. conclusion

This research explores the transmission dynamics of Maize Foliar Disease (MFD) in maize plants, considering the interactions between crops, and fungal populations caus-

ing the disease. We analyzed both the disease-free equilibrium (DFE) and the disease persistence equilibrium (DPE) points, deriving criteria to ensure their stability. The basic reproduction number \mathcal{R}_0 was calculated. When $\mathcal{R}_0 < 1$, the DFE is globally asymptotically stable, while when $\mathcal{R}_0 > 1$, the DPE is globally asymptotically stable.

The findings underscored the critical influence of recruitment rates (Λ_1, Λ_2) and transmission rates (β_1, β_2) on the propagation of MFD among maize plants. Also the study highlights the MFD-induced death rate (δ) in maize plants as the parameter with the negative sensitivity. This underscores the significant influence of effectively managing the MFD-induced mortality rate on reducing disease prevalence. Numerical simulation indicated that, effective implementation of disease management strategies, such as weed control and the use of disease-resistant maize varieties such as NK-corn hybrids, Pioneer brand maize hybrids, and DEKALB maize varieties, could potentially mitigate the spread of MFD.

However, limitations exist, including dependency on available data for parameter estimation as also found by Mgandu et al. [45] and potential disparities between assumptions and real-world complexities, such as variations in disease dynamics over time and across different geographical locations the limitation which was also supported by Aloyce et al. [47] and Liu and Huang [36]. The study can exhibit considerable variability over time and across different geographical regions, as demonstrated in studies by Sk et al. [48], Zhang [49] and Seidu [54].

Despite these limitations, the findings highlight the critical role of integrated disease management strategies in controlling Maize Foliar Disease (MFD) transmission and safeguarding maize crop production. Future research should explore the effectiveness of various disease management strategies, including preventive measures and treatments, that would help guide agricultural practices and policy development to mitigate the impact of MFD on maize production.

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Availability of Data

All data generated for this manuscript are listed in the references.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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