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Generalized Mittag-Leffler Function: Properties in Fractional Calculus and Integral Transforms

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Abstract

In this work, we introduce a new generalized Mittag-Leffler function defined via the extended Beta function and establish its integral and differential representations. Several fundamental properties are derived, including differentiation formulas and the Beta, Laplace, and Mellin transforms, together with relationships to the Wright function and generalized hypergeometric functions. Furthermore, the behavior of the associated Riemann-Liouville fractional integrals and derivatives of the proposed function is investigated. A number of interesting special cases are also presented to illustrate the generality and unifying nature of the main results. In the final section, we discuss potential applications of the newly defined Mittag-Leffler function, demonstrate its use in solving a fractional kinetic equation, and outline possible directions for future research.

Keywords: Extended Beta function, Mittag-Leffler function, Laplace transform, fractional calculus operators, fractional kinetic equation.
2010 MSC: 26A33, 33E12, 33C05, 33C15, 33C20.

1. Introduction

Several interesting generalizations of the familiar Euler-Gamma function $\Gamma(z)$, Euler-Beta function $B(x,y)$, the Gauss hypergeometric functions ${}_2F_1$ and the generalized hypergeometric functions ${}_pF_q$ with p numerator and q denominator were studied and investigated by various authors (see, for example, [1, 2, 3, 4, 5, 6, 7] and the references cited in each of these papers). For example, for an appropriately-bounded sequence $\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}$ of arbitrary (real or complex) numbers, Srivastava *et al.* (see[8], p. 243, Equation (2.1)) recently considered the function:

$$\Theta(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};z)=\begin{cases}\sum_{n=0}^{\infty}\mathcal{K}_l\frac{z^l}{l!} & (z<R;0<R<\infty;\mathcal{K}_0=1),\\ \\ \mathfrak{M}_0 z^w\exp(z)[1+O(\frac{1}{z})] & (\mathrm{Re}(z)\to\infty;\mathfrak{M}_0>0;w\in\mathbb{C}).\end{cases}\tag{1.1}$$

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for some suitable constants $\mathfrak{M}_0$ and w depending essentially on the sequence $\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}$. In terms of the function $\Theta(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};z)$ defined by Equation (1.1), Srivastava *et al.* [8], introduced far-reaching generalizations of the extended Gamma function, extended Beta function and the extended Gauss hypergeometric function by:

$$\Gamma_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(z)=\int_0^\infty t^{z-1}\Theta\left(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};-t-\frac{p}{t}\right)dt, \tag{1.2}$$
$$(\mathrm{Re}(p)\geqslant 0,\mathrm{Re}(z)>0),$$

$$B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}\Theta\left(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\frac{-p}{t(1-t)}\right)dt, \tag{1.3}$$
$$(\mathrm{Re}(p)\geqslant 0,\mathrm{Re}(x)>0,\mathrm{Re}(y)>0),$$

$$\mathcal{F}_p^{\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(a,b;c;z)=\sum_{n=0}^\infty (a)_n\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!}, \tag{1.4}$$
$$(|z|<1;p\geqslant 0;\mathrm{Re}(c)>\mathrm{Re}(b)>0),$$

respectively, provided that the defining integrals in the definitions (Equations (1.2)-(1.4)) exist.
For various special choices of the sequence $\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}$, the definition in Equations (1.2)-(1.4) would reduce to (known or new) extensions of the Gamma, Beta and hypergeometric functions. In particular, if we set:

$$\mathcal{K}_l=\frac{(\alpha)_l}{(\beta)_l}\ (l\in\mathbb{N}_0), \tag{1.5}$$

the definition (Equations (1.2)-(1.4)) immediately reduces to the extended Gamma function $\Gamma_p^{(\alpha,\beta)}(z)$, the extended Beta function $B_p^{(\alpha,\beta)}(x,y)$ and the extended hypergeometric function $F_p^{(\alpha,\beta)}(a,b;c;z)$ introduced by Özergin *et al.* [9]:

$$\Gamma_p^{(\alpha,\beta)}(z)=\int_0^\infty t^{z-1}{}_1F_1\left(\alpha;\beta;-t-\frac{p}{t}\right)dt, \tag{1.6}$$
$$(\mathrm{Re}(p)\geqslant 0,\mathrm{Re}(z)>0,\mathrm{Re}(\alpha)>0,\mathrm{Re}(\beta)>0),$$

$$B_p^{(\alpha,\beta)}(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}{}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right)dt, \tag{1.7}$$
$$\mathrm{Re}(p)\geqslant 0,\mathrm{Re}(x)>0,\mathrm{Re}(y)>0),$$

$$F_p^{(\alpha,\beta)}(a,b;c;z)=\sum_{n=0}^\infty (a)_n\frac{B_p^{(\alpha,\beta)}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!}, \tag{1.8}$$
$$(|z|<1;p\geqslant 0;\mathrm{Re}(\alpha)>0,\mathrm{Re}(\beta)>0;\mathrm{Re}(c)>\mathrm{Re}(b)>0),$$

respectively. Furthermore, for the sequence $\mathcal{K}_l=1$, the definition (Equations (1.2)-(1.4)) reduces immediately to the generalized Gamma function, extended Beta function and extended Gauss hypergeometric function studied earlier by Chaudhry and Zubair ([1], p.

9, Equation (1.66)), Chaudhry *et al.* [2] and Chaudhry *et al.* [4]:

$$\Gamma_p(z) = \int_0^\infty t^{z-1} \exp\left(-t - \frac{p}{t}\right) dt, \tag{1.9}$$

$$(\mathrm{Re}(p) \geqslant 0, \mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0; z \in \mathbb{C}),$$

$$B_p(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1} \exp\left(\alpha; -\frac{p}{t(1-t)}\right) dt, \quad (\mathrm{Re}(p) \geqslant 0) \tag{1.10}$$

$$F_p(a,b;c;z) = \sum_{n=0}^{\infty} (a)_n \frac{B_p(b+n, c-b)}{B(b,c-b)} \frac{z^n}{n!}, \tag{1..11}$$

$$(|z| < 1; p \geqslant 0; \mathrm{Re}(c) > \mathrm{Re}(b) > 0),$$

respectively. For $p = 0$ and the sequence $\mathcal{K}_l = 1$, the definitions (Equations (1.2)-(1.4)) would reduce immediately to classical Gamma, Beta and Gauss hypergeometric functions (see, for details, [10, 11, 12], respectively.
The classical Mittag-Leffler function [13] is defined by:

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)}, \ (z \in \mathbb{C}; \mathrm{Re}(\alpha) > 0). \tag{1.12}$$

The Mittag-Leffler function is a direct generalization of $\exp(z)$ in which $\alpha = 1$. Mittag-Leffler function naturally occurs as the solution of fractional order differential equation or fractional order integral equations.

Wiman [14] suggested the generalization of $E_{\alpha,\beta}(z)$ for $z, \alpha, \beta \in \mathbb{C}; \mathrm{Re}(\alpha) > 0$, $\mathrm{Re}(\beta) > 0$ as

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)}, \tag{1.13}$$

which is known as Wimanś function or the generalized Mittag-Leffler function with two parameter.

Prabhakar [15] introduced the function $E^\gamma_{\alpha,\beta}(z)$ in the form

$$E^\gamma_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n + \beta)} \frac{z^n}{n!}, \ (\alpha, \beta, \gamma \in \mathbb{C}; \mathrm{Re}\{\alpha, \beta, \gamma\} > 0), \tag{1.14}$$

where $(\gamma)_n$ is the Pochhammer symbol defined as
$(\gamma)_n = \frac{\Gamma(\gamma+n)}{\Gamma(\gamma)} = \begin{cases} 1 & (n = 0), \\ \gamma(\gamma+1)\ldots(\gamma+n-1) & (n = 1, 2, \ldots). \end{cases}$

A next extension was introduced by Salim [12]:

$$E^{\gamma,c}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n + \beta)(c)_n} z^n, \tag{1.15}$$

$$(z, \alpha, \beta, \gamma, c \in \mathbb{C}; \ \min\{\mathrm{Re}(\alpha), \mathrm{Re}(\beta), \mathrm{Re}(c)\} > 0).$$

The study of generalized Mittag-Leffler functions remains highly relevant due to their intrinsic connection to solutions of fractional differential equations. Recent advances in the theory and numerical analysis of generalized fractional operators, such as the Caputo-Katugampola derivative [16], further underscore the need for a comprehensive understanding of the special functions that arise in this context. The generalized function introduced in this work provides a potential tool for constructing analytical solutions to such modern fractional problems. Also, the article [17] focuses on the solution formulas, existence, and uniqueness of two types of Cauchy problems for impulsive fractional differential equations involving the Atangana-Baleanu-Caputo (ABC) derivative with a nonsingular Mittag-Leffler kernel. Further, various authors studied and investigated generalized Mittag-Leffler functions (see, for details, [18, 9, 19, 20, 13, 10, 21, 22]. Motivated essentially by the demonstrated potential for applications of these extended hypergeometric functions, Parmar [23] investigated and extended the generalized Mittag-Leffler function (Equations (1.14) by means of the extended Beta function $B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(x,y)$ defined by Equation (1.3)as follows:

$$E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(z;p)=\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,1-\gamma)}{B(\gamma,1-\gamma)}\frac{z^n}{\Gamma(\alpha n+\beta)},\tag{1.16}$$

$$(z,\alpha,\beta,\gamma\in\mathbb{C};\ \operatorname{Re}(\alpha)>0,\operatorname{Re}(\beta)>0,\operatorname{Re}(\gamma)>1;p\geqslant 0).$$

The special case of Equation (1.16), set the sequence $\mathcal{K}_l=\frac{(\rho)_l}{(\sigma)_l}$, yields the known definition of Parmar [23]:

$$E_{\alpha,\beta}^{(\rho,\sigma;\gamma)}(z;p)=\sum_{n=0}^{\infty}\frac{B_p^{(\rho,\sigma)}(\gamma+n,1-\gamma)}{B(\gamma,1-\gamma)}\frac{z^n}{\Gamma(\alpha n+\beta)},\tag{1.17}$$

$$(z,\alpha,\beta,\gamma\in\mathbb{C};\ \operatorname{Re}(\alpha)>0,\operatorname{Re}(\beta)>0,\operatorname{Re}(\gamma)>1;p\geqslant 0).$$

Another recent extension is defined by Padma *et al.* in [24]:

$$E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)=\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{z^n}{\Gamma(\alpha n+\beta)},\tag{1.18}$$

$$(z,\alpha,\beta,\gamma,c\in\mathbb{C};\ \operatorname{Re}(\alpha)>0,\operatorname{Re}(\beta)>0,\operatorname{Re}(c)>0,\operatorname{Re}(\gamma)>1;p\geqslant 0).$$

The special case of Equation (1.17), set the sequence $\mathcal{K}_l=\frac{(\rho)_l}{(\sigma)_l}$, yields the known definition of Padma *et al.* [24]:

$$E_{\alpha,\beta}^{(\rho,\sigma;\gamma,c)}(z;p)=\sum_{n=0}^{\infty}\frac{B_p^{(\rho,\sigma)}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{z^n}{\Gamma(\alpha n+\beta)},\tag{1.19}$$

$$(z,\alpha,\beta,\gamma,c\in\mathbb{C};\ \operatorname{Re}(\alpha)>0,\operatorname{Re}(\beta)>0,\operatorname{Re}(c)>0,\operatorname{Re}(\gamma)>1;p\geqslant 0).$$

2. Extended Mittag-Leffler function

We define an extended Mittag-Leffler function as follows:

Definition 2.1. For $z,\alpha,\beta,\gamma,a,c \in \mathbb{C};\ \mathrm{Re}\{\alpha,\beta,a,c\} > 0, \mathrm{Re}(\gamma) > 1; p \geqslant 0$, we define the $E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)$ function in the following form:

$$E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p) = \sum_{n=0}^{\infty} \frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!}. \tag{2.1}$$

The suggested function in Eq. (2.1) is motivated by the fact that it unifies and extends several previously introduced Mittag-Leffler-type families through the inclusion of an additional Pochhammer (hypergeometric) weight $(a)_n/n!$ in the coefficients, while retaining the flexible extended Beta factor $B_p^{(\{\mathcal{K}_l\})}$. For $a=1$, the ratio $(a)_n/n! = (1)_n/n! = 1$, and the series reduces to the earlier Padma-Parmar type extension; other special choices of the sequence $\{\mathcal{K}_l\}$ or of p recover classical and contemporary Mittag-Leffler, Wright, and hypergeometric-type functions. The additional parameter a therefore provides a controlled bridge between Mittag-Leffler behaviour and hypergeometric Wright-type growth, increasing modeling flexibility and enabling new integral representations, transform pairs, and fractional differential operator solutions which are useful in fractional calculus, anomalous diffusion, viscoelasticity, and related applied fields.

Region of convergence:. We examine the power series

$$E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\};\gamma,c)}(z;p) = \sum_{n=0}^{\infty} \underbrace{\frac{B_p^{(\{\mathcal{K}_l\})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{1}{n!}}_{=:u_n} z^n.$$

The behaviour of the coefficients u_n as $n \to \infty$ determines the region of convergence. Using standard asymptotics one has

$$\frac{(a)_n}{n!} = \frac{\Gamma(a+n)}{\Gamma(a)\,n!} \sim \frac{n^{\,a-1}}{\Gamma(a)} \qquad (n \to \infty),$$

while Stirling's formula gives, for $\mathfrak{R}(\alpha) > 0$,

$$\Gamma(\alpha n+\beta) \sim C\,(\alpha n)^{\alpha n+\beta-\frac{1}{2}} e^{-\alpha n},$$

so that $1/\Gamma(\alpha n+\beta)$ decays super-factorially (much faster than any geometric rate). Thus any polynomial (or even any exponential) growth of the extended Beta factor $B_p^{(\{\mathcal{K}_l\})}(\gamma+n,c-\gamma)$ is dominated by the denominator $\Gamma(\alpha n+\beta)$.

Consequently one obtains the following sufficient criterion.

Lemma 2.1. *Suppose $\mathfrak{R}(\alpha) > 0$ and there exist constants $C > 0$ and $m \geqslant 0$ such that*

$$\left|B_p^{(\{\mathcal{K}_l\})}(\gamma+n,c-\gamma)\right| \leqslant C\,n^m \qquad (n \gg 1).$$

Then the series in (2.1) converges for every $z \in \mathbb{C}$; i.e. the function is entire.

Proof. With the polynomial bound above and the asymptotic $(a)_n/n! \sim n^{a-1}/\Gamma(a)$, we have for large n

$$|u_n| \leqslant C' n^{m+a-1} \left|\Gamma(\alpha n+\beta)\right|^{-1},$$

and the right-hand side tends to zero faster than any geometric sequence because of Stirling's estimate for $\Gamma(\alpha n+\beta)$. Hence the radius of convergence is infinite.

Observations:

1. The polynomial-growth condition on $B_p^{(\{\mathcal{K}_l\})}(\gamma+n, c-\gamma)$ is satisfied in the classical (non-extended) case since $B(\gamma+n, c-\gamma) = \dfrac{\Gamma(\gamma+n)\Gamma(c-\gamma)}{\Gamma(c+n)} = O(n^{\gamma-c})$.
2. If $B_p^{(\{\mathcal{K}_l\})}(\gamma+n, c-\gamma)$ exhibits at most exponential growth, i.e. $|B_p(\gamma+n, c-\gamma)| = O(e^{\kappa n})$ for some $\kappa \geqslant 0$, the series still converges for all $z \in \mathbb{C}$ because the super-factorial decay of $1/\Gamma(\alpha n+\beta)$ dominates any fixed exponential. Thus entireness persists under very mild growth of the extended Beta factor.
3. If one allows pathological choices of the sequence $\{\mathcal{K}_l\}$ for which $B_p^{(\{\mathcal{K}_l\})}(\gamma+n, c-\gamma)$ grows faster than $\Gamma(\alpha n+\beta)$ (this would be unusual and should be excluded by hypotheses on Θ), then the radius of convergence may be finite; such exceptional behaviour should be checked case by case.

Remark 2.1. The special case of Equation (2.1) when we set the sequence $\mathcal{K}_l = \frac{(\rho)_l}{(\sigma)_l}$ $(l \in \mathbb{N}_0)$, yields another form as follows:

$$E_{\alpha,\beta;a}^{(\rho,\sigma;\gamma,c)}(z;p) = \sum_{n=0}^{\infty} \frac{B_p^{(\rho,\sigma)}(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!}, \tag{2.2}$$

$$(z, \alpha, \beta, \gamma, a, c \in \mathbb{C};\ \operatorname{Re}(\alpha) > 0, \operatorname{Re}(\beta) > 0, \operatorname{Re}(c) > \operatorname{Re}(\gamma) > 0; p \geqslant 0).$$

Again, setting $p = 0$, yields second form of the generalized Mittag-Leffler function:

$$E_{\alpha,\beta}^{(\gamma,a;c)}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n (a)_n}{\Gamma(\alpha n+\beta)(c)_n} \frac{z^n}{n!}, \tag{2.3}$$

$$(z, \alpha, \beta, \gamma, a, c \in \mathbb{C};\ \min\{\operatorname{Re}(\alpha), \operatorname{Re}(\beta), \operatorname{Re}(c)\} > 0).$$

Remark 2.2. Among many other things, the following special cases of the extended Mittag-Leffler function $E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)$ in (2.1) are considered in our present investigation.
Case 1. The special cases, first, For $a = c = 1$ in (2.1), (2.2) and (2.3) reduces to (1.16), (1.17) and (1.14) respectively.
For $\alpha = 0; \beta = 1$, in (2.1), (2.2) reduces to (1.4), (1.8) respectively.
Case 2. The special case of Equation (2.1) for $a = c$ and the sequence $\mathcal{K}_l = 1 (l \in \mathbb{N})$, yields the known definition of Özarslan and Yilmaz [25]:

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p) = \sum_{n=0}^{\infty} \frac{B_p(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)} \frac{(c)_n}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!}, \tag{2.4}$$

$$(z, \alpha, \beta, \gamma, c \in \mathbb{C};\ \operatorname{Re}(\alpha) > 0, \operatorname{Re}(\beta) > 0, \operatorname{Re}(c) > \operatorname{Re}(\gamma) > 0; p \geqslant 0).$$

Case 3. The special case $a = 1$ in (2.1), (2.2) and (2.3) reduces to (1.18), (1.19) and (1.15) respectively.
Case 4. The special case $a = 1$ and the sequence $\mathcal{K}_l = \frac{(\rho)_l}{(\sigma)_l}$ $(l \in \mathbb{N}_0)$, in (2.1) reduces to the extended confluent hypergeometric function of Özergin *et al.* [6]:

$$\Phi_p^{(\alpha,\beta)}(\gamma; c; z) = \sum_{n=0}^{\infty} \frac{B_p^{(\alpha,\beta)}(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)} \frac{z^n}{n!}, \tag{2.5}$$

$$(p \geqslant 0; \mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0; \mathrm{Re}(c) > \mathrm{Re}(\gamma) > 0).$$

Case 5. The special case $\alpha = \beta$ and the sequence $\mathcal{K}_l = 1$ $(l \in \mathbb{N})$, in (2.1) reduces to extended Gauss hypergeometric function (1.11) defined by Chaudhry *et al.* [4]. Again, the sequence $\mathcal{K}_l = 1$ $(l \in \mathbb{N})$, and $\alpha = \beta = a = 1$ in (2.1) reduces to the extended confluent hypergeometric function of Chaudhry *et al.* [4]:

$$\Phi_p(\gamma; c; z) = \sum_{n=0}^{\infty} \frac{B_p(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)} \frac{z^n}{n!}, \quad (p \geqslant 0; \mathrm{Re}(c) > \mathrm{Re}(\gamma) > 0). \tag{2.6}$$

Remark 2.3. Regarding the numerical and computational aspects for practical applications, the series in (2.1) can be computed directly for small to moderate $|z|$. For larger $|z|$, convergence acceleration techniques or integral representations via the extended Beta function $B_p^{(\{\mathcal{K}_l\})}$ can improve efficiency. Recursion relations and asymptotic expansions also provide useful tools for numerical implementation, as discussed in standard references on Mittag-Leffler and hypergeometric-type functions [26, 27].

For our purpose, we recall the extension of Wright hypergeometric function is defined by Agarwal *et al.*, ([28], Eq. 1.13) for $z, \gamma \in \mathbb{C}; \mathrm{Re}(c) > \mathrm{Re}(\gamma) > 0; p \geqslant 0)$ as fowolls:

$$\begin{aligned} {}_{m+1}\psi_{n+1}^{((\mathcal{K}_l)_{l\in\mathbb{N}_0})}(z;p) &= {}_{m+1}\psi_{n+1}^{((\mathcal{K}_l)_{l\in\mathbb{N}_0})}\left[\begin{array}{c|c} (a_i,\alpha_i)_{1,m}, \quad (\gamma,1) & \\ & (z;p) \\ (b_i,\beta_i)_{1,n}, \quad (c,1) & \end{array}\right] \\ &= \frac{1}{\Gamma(c-\gamma)} \sum_{k=0}^{\infty} B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n, c-\gamma) \frac{\prod_{i=1}^{m}\Gamma(a_i+k\alpha_i)}{\prod_{i=1}^{n}\Gamma(b_i+k\beta_i)} \frac{z^k}{k!}. \end{aligned} \tag{2.7}$$

Corollary 2.1. we have

$$E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(\gamma)} {}_2\psi_2^{((\mathcal{K}_l)_{l\in\mathbb{N}_0})}\left[\begin{array}{c|c} (a,1), \quad (\gamma,1) & \\ & (z;p) \\ (\beta,\alpha), \quad (c,1) & \end{array}\right]. \tag{2.8}$$

3. Basic Properties of $E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)$

In this section, we obtain certain basic properties, including the differentiation formula and the integral property of the extended Mittag-Leffler function in Equation (2.1).

Theorem 3.1. The following differentiation formula for the extended Mittag-Leffler function in Equation (2.1) holds true:

$$E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)=\beta E_{\alpha,\beta+1;c}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,a)}(z;p)+\alpha z\frac{d}{dz}E_{\alpha,\beta+1;c}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,a)}(z;p), \tag{3.1}$$

$$(z,\alpha,\beta,\gamma,a,c\in\mathbb{C};\ \mathrm{Re}\{\alpha)\beta,a,c\}>0,\mathrm{Re}(\gamma)>1;p\geqslant 0).$$

In particular, we have:

$$E_{\alpha,\beta}^{(\gamma,a;c)}(z)=\beta E_{\alpha,\beta+1}^{(\gamma,a;c)}(z)+\alpha z\frac{d}{dz}E_{\alpha,\beta+1}^{(\gamma,a;c)}(z). \tag{3.2}$$

Proof. Using the definition (Equation (2.1)) in right-hand side of Equation (3.1), we have:

$$\begin{aligned}
&\beta E_{\alpha,\beta+1;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)+\alpha z\frac{d}{dz}E_{\alpha,\beta+1;c}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,a)}(z;p)\\
&=\beta E_{\alpha,\beta+1;c}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,a)}(z;p)+\alpha z\frac{d}{dz}\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta+1)}\frac{z^n}{n!}\\
&=\beta E_{\alpha,\beta+1;c}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,a)}(z;p)+\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta+1)}\frac{\alpha n z^n}{n!}\\
&=\beta E_{\alpha,\beta+1;c}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,a)}(z;p)+\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta+1)}\frac{(\alpha n+\beta-\beta)z^n}{n!}\\
&=E_{\alpha,\beta;c}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,a)}(z;p).
\end{aligned}$$

The relation Equation (3.2) follows from Equation (3.1) when $p=0$ or for $\mathcal{K}_l=0(l\in\mathbb{N})$.
□

Obviously, for $a=c=1$, (3.1) and (3.2) would immediately reduces to the known results of Parmar ([23], p.1073 , Eq. (19), (20)):

$$E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(z;p)=\beta E_{\alpha,\beta+1}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(z;p)+\alpha z\frac{d}{dz}E_{\alpha,\beta+1}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(z;p), \tag{3.3}$$

and

$$E_{\alpha,\beta}^{(\gamma)}(z)=\beta E_{\alpha,\beta+1}^{(\gamma)}(z)+\alpha z\frac{d}{dz}E_{\alpha,\beta+1}^{(\gamma)}(z), \tag{3.4}$$

respectively.
For $a=1$, (3.1) reduces to the formula (see (Padma *et al.* [24]):

$$E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)=\beta E_{\alpha,\beta+1}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)+\alpha z\frac{d}{dz}E_{\alpha,\beta+1}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p). \tag{3.5}$$

Theorem 3.2. Let $\alpha,\beta,\gamma,a,c\in\mathbb{C}$ be such that $\{\Re(\alpha),\Re(\beta),\Re(a),\Re(c)>0,\ \Re(\gamma)\}>1$, and $p\geqslant 0$. Then, we can express the function in the following integral representation:

$$E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)$$

$$= \frac{1}{B(\gamma, c-\gamma)} \int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1}\Theta\left(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}; \frac{-p}{t(1-t)}\right) E_{\alpha,\beta}^{(a)}(tz)\,dt. \tag{3.6}$$

Proof. Using Eq. (1.3) in Eq. (2.1), we obtain

$$\begin{aligned} &E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p) \\ &= \sum_{n=0}^{\infty}\left\{\frac{1}{B(\gamma, c-\gamma)} \int_0^1 t^{\gamma+n-1}(1-t)^{c-\gamma-1}\Theta\left(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}; \frac{-p}{t(1-t)}\right) dt\right\} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!} \\ &= \frac{1}{B(\gamma, c-\gamma)} \int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1}\Theta\left(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}; \frac{-p}{t(1-t)}\right) \sum_{n=0}^{\infty} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{(tz)^n}{n!}\,dt,. \end{aligned} \tag{3.7}$$

Using Eq. (1.14) in Eq. (3.7), we obtain the formula (3.6) immediately. □
In its special cases when $p = 0$, (3.6) reduces to the formula:

$$E_{\alpha,\beta}^{(\gamma,a;c)}(z) = \frac{1}{B(\gamma, c-\gamma)} \int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1} E_{\alpha,\beta}^{(a)}(tz)\,dt, \tag{3.8}$$

For $a = 1$, (3.6) reduces to the formula (see Padma *et al.* [24]):

$$\begin{aligned} &E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p) \\ &= \frac{1}{B(\gamma, c-\gamma)} \int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1}\Theta\left(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}; \frac{-p}{t(1-t)}\right) E_{\alpha,\beta}(tz)\,dt, \end{aligned} \tag{3.9}$$

Corollary 3.1. Taking $t = \frac{u}{u+1}$ in Theorem 3.2., we obtain

$$\begin{aligned} &E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p) \\ &= \frac{1}{B(\gamma, c-\gamma)} \int_0^\infty \frac{u^{\gamma-1}}{(1+u)^c}\Theta\left(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}; \frac{-p(1+u)^2}{u}\right) E_{\alpha,\beta}^{(a)}\left(\frac{zu}{1+u}\right) du, \end{aligned} \tag{3.10}$$

Corollary 3.2. Taking $t = \sin^2\theta$ in Theorem 3.2., we obtain

$$\begin{aligned} &E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p) \\ &= \frac{2}{B(\gamma, c-\gamma)} \int_0^{\frac{\pi}{2}} \sin^{2\gamma-1}\theta\cos^{2c-2\gamma-1}\theta\Theta\left(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}; \frac{-p}{\sin^2\theta\cos^2\theta}\right) E_{\alpha,\beta}^{(a)}(z\sin^2\theta)\,d\theta, \end{aligned} \tag{3.11}$$

Corollary 3.3. The following integral belongings to Eq.(3.1) follow as:

$$\int_0^z t^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\lambda t^\alpha;p)\,dt = z^\beta E_{\alpha,\beta+1;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\lambda z^\alpha;p). \tag{3.12}$$

In particular, for $p = 0$ we have:

$$\int_0^z t^{\beta-1} E_{\alpha,\beta}^{(\gamma;c,a)}(\lambda t^\alpha)\, dt = z^\beta E_{\alpha,\beta+1}^{(\gamma;c,a)}(\lambda z^\alpha). \tag{3.13}$$

In its special cases when $a = 1$, (3.9) reduces to the formula (see Padma *et al.* [24]):

$$\int_0^z t^{\beta-1} E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\lambda t^\alpha; p)\, dt = z^\beta E_{\alpha,\beta+1}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\lambda z^\alpha; p). \tag{3.14}$$

For $a = c = 1$, (3.9) and (3.10) reduces to the formulas [23]:

$$\int_0^z t^{\beta-1} E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(\lambda t^\alpha; p)\, dt = z^\beta E_{\alpha,\beta+1}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(\lambda z^\alpha; p), \tag{3.15}$$

$$\int_0^z t^{\beta-1} E_{\alpha,\beta}^{(\gamma)}(\lambda t^\alpha)\, dt = z^\beta E_{\alpha,\beta+1}^{(\gamma)}(\lambda z^\alpha), \tag{3.16}$$

respectively.

Theorem 3.3. The following derivative formulas for the extended Mittag-Leffler function in Equation (2.1) are satisfied:

$$\left(\frac{d}{dz}\right)^k \left[z^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\lambda z^\alpha; p)\right] = z^{\beta-k-1} E_{\alpha,\beta-k;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\lambda z^\alpha; p),\ (\mathrm{Re}(\beta-k) > 0, m \in \mathbb{N}). \tag{3.17}$$

In particular, we have:

$$\left(\frac{d}{dz}\right)^k \left[z^{\beta-1} E_{\alpha,\beta}^{(\gamma;c,a)}(\lambda z^\alpha)\right] = z^{\beta-k-1} E_{\alpha,\beta-k}^{(\gamma;c,a)}(\lambda z^\alpha). \tag{3.18}$$

Proof. Using Equation (2.1) and employing term-wise differentiation m times on the left-hand side of Equation (3.17) under the summation sign, which is possible in accordance with the uniform convergence of the series in Equation (2.1), we get:

$$\begin{aligned}
&\left(\frac{d}{dz}\right)^k \left[z^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\lambda z^\alpha; p)\right] \\
&= \sum_{n=0}^{\infty} \frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{\lambda^n}{n!} \left(\frac{d}{dz}\right)^k [z^{\alpha n+\beta-1}] \\
&= \sum_{n=0}^{\infty} \frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{\lambda^n}{n!} \frac{\Gamma(\alpha n+\beta)}{\Gamma(\alpha n+\beta-k)} z^{\alpha n+\beta-k-1} \\
&= z^{\beta-k-1} E_{\alpha,\beta-k;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\lambda z^\alpha; p).
\end{aligned}$$

The special cases of Equation (3.17) when $p = 0$ or for $\mathcal{K}_l = 0 (l \in \mathbb{N})$ are easily seen to yield Equation (3.18). □

Obviously, for $a = c = 1$, (3.17) and (3.18) would immediately reduces to the known results of Parmar ([23], p.1073 , Eq. (21), (22)):

$$\left(\frac{d}{dz}\right)^k \left[z^{\beta-1} E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(\lambda z^\alpha;p)\right] = z^{\beta-k-1} E_{\alpha,\beta-k}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(\lambda z^\alpha;p), \tag{3.19}$$

and

$$\left(\frac{d}{dz}\right)^k \left[z^{\beta-1} E_{\alpha,\beta}^{(\gamma)}(\lambda z^\alpha)\right] = z^{\beta-k-1} E_{\alpha,\beta-k}^{(\gamma)}(\lambda z^\alpha), \tag{3.20}$$

respectively.

4. Integral transform of $E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)$

In this section, we obtain the Laplace transform, Mellin transform representations and the Euler-Beta transform, alternatively called the Erdélyi-Kober fractional integral for the extended Mittag-Leffler function $E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)$, in Equation (2.1) as follows.

4.1. Laplace Transform

The Laplace transform (see, e.g., [29]) of the function $f(z)$ is defined, as usual, by:

$$\mathcal{L}\{f(z)\} = \int_0^\infty e^{-sz} f(z)\,dz, \quad (\mathrm{Re}(s) > 0). \tag{4.1}$$

Theorem 4.1. The following Laplace transform representation for the extended generalized Mittag-Leffler function in Equation (2.1) holds true:

$$\mathcal{L}\left\{z^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^\alpha;p)\right\} = \frac{1}{s^\beta} \mathcal{F}_p^{\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}\left(a,\gamma;c;\frac{x}{s^\alpha}\right). \tag{4.2}$$

In particular, we have:

$$\mathcal{L}\left\{z^{\beta-1} E_{\alpha,\beta}^{(\gamma;c,a)}(xz^\alpha)\right\} = \frac{1}{s^\beta}\, {}_2F_1\left(a,\gamma;c;\frac{x}{s^\alpha}\right), \tag{4.3}$$

Proof. Using the definition (Equation (4.1)) of the Laplace transform, we find from

Equation (2.1):

$$\mathcal{L}\left\{z^{\beta-1}E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^\alpha;p)\right\}=\int_0^\infty z^{\beta-1}e^{-sz}E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^\alpha;p)\,dz$$

$$=\int_0^\infty z^{\beta-1}e^{-sz}\left(\sum_{n=0}^\infty\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta)}\frac{x^nz^{\alpha n}}{n!}\right)dz$$

$$=\sum_{n=0}^\infty\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta)}\frac{x^n}{n!}\int_0^\infty z^{\alpha n+\beta-1}e^{-sz}\,dz$$

$$=\sum_{n=0}^\infty\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta)}\frac{x^n}{n!}\frac{\Gamma(\alpha n+\beta)}{s^{\alpha n+\beta}}$$

$$=\frac{1}{s^\beta}\sum_{n=0}^\infty(a)_n\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(\frac{x}{s^\alpha})^n}{n!}.$$

Now, using the definition (Equation (1.4)) to express the involved sum as an extended hypergeometric function, we are led to the desired result. The special cases of Equation (4.2) when $p=0$ or for $\mathcal{K}_l=0(l\in\mathbb{N})$ are easily seen to yield Equation (4.3). □
Obviously, for $a=c=1$, (4.2) and (4.3) reduces to the known Laplace transform of the generalized Mittag-Leffler functions ([23], p.1074 , Eq. (26)) and [15], p.8, Eq. (2.5))

$$\mathcal{L}\left\{z^{\beta-1}E_{\alpha,\beta}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma)}(xz^\alpha;p)\right\}=\frac{1}{s^\beta}\mathcal{F}_p^{\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}\left(1,\gamma;1;\frac{x}{s^\alpha}\right),\tag{4.4}$$

and

$$\mathcal{L}\left\{z^{\beta-1}E_{\alpha,\beta}^{(\gamma)}(xz^\alpha)\right\}=\frac{1}{s^\beta}\left(1-\frac{x}{s^\alpha}\right)^{-\gamma},\tag{4.5}$$

respectively.

4.2. Eluer-Beta Transform

The Euler-Beta transform [29], alternatively called the Erdélyi-Kober fractional integral of the function $f(z)$, is defined, as usual, by:

$$B\{f(z);r,s\}=\int_0^1 z^{r-1}(1-z)^{s-1}f(z)\,dz.\tag{4.6}$$

Theorem 4.2. The following Euler-Beta transform or Erdélyi-Kober fractional integral representation for the extended Mittag-Leffler function in Equation (2.1) holds true:

$$B\{E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^\alpha;p);\beta,b\}=\Gamma(b)E_{\alpha,\beta+b;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(x;p),\tag{4.7}$$

$$B\{E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^\rho;p);r,s\}$$
$$=\frac{\Gamma(c)\Gamma(s)}{\Gamma(a)\Gamma(\gamma)}{}_3\psi_3^{((\mathcal{K}_l)_{l\in\mathbb{N}_0})}\left[\begin{array}{c|c}(a,1),\ (r,\rho),\ \ (\gamma,1) & \\ & (x;p)\\ (\beta,\alpha),\ (r+s,\rho),\ \ (c,1) & \end{array}\right].\tag{4.8}$$

Proof. Using the definition (Equation (4.6)) of the Euler-Beta transform, we find from Equation (2.1):

$$\begin{aligned}
&B\{E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^{\alpha};p);\beta,b\} = \int_0^1 z^{\beta-1}(1-z)^{b-1}E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^{\alpha};p)\,dz\\
&= \sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta)}\frac{x^n}{n!}\int_0^1 z^{\alpha n+\beta-1}(1-z)^{b-1}\,dz\\
&= \sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta)}\frac{x^n}{n!}\frac{\Gamma(\alpha n+\beta)\Gamma(b)}{\Gamma(\alpha n+\beta+b)}\\
&= \Gamma(b)\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta+b)}\frac{x^n}{n!}.
\end{aligned}$$

Using the definition (Equation (2.1)), we get the desired representation Equation (4.7). Again, using (4.6) and (2.1), it gives

$$\begin{aligned}
&B\{E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^{\rho};p):r,s\} = \int_0^1 z^{r-1}(1-z)^{s-1}E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^{\rho};p)\,dz\\
&= \sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{\Gamma(\alpha n+\beta)}\frac{x^n}{n!}\int_0^1 z^{r+\rho n-1}(1-z)^{s-1}\,dz\\
&= \frac{\Gamma(c)}{\Gamma(a)\Gamma(\gamma)}\frac{1}{\Gamma(c-\gamma)}\sum_{n=0}^{\infty}B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)\frac{\Gamma(a+n)}{\Gamma(\alpha n+\beta)}\frac{x^n}{n!}B(r+\rho n,s)\\
&= \frac{\Gamma(c)\Gamma(s)}{\Gamma(a)\Gamma(\gamma)}\frac{1}{\Gamma(c-\gamma)}\sum_{n=0}^{\infty}B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)\frac{\Gamma(a+n)\Gamma(r+\rho n)}{\Gamma(\alpha n+\beta)\Gamma(r+s+\rho n)}\frac{x^n}{n!}.
\end{aligned}$$

In accordance with the definition of (2.8), we obtain the desired result (4.8). □

4.3. Whittaker Transform

The following integral involving the Whittakar function

$$\int_0^{\infty} z^{\nu-1}e^{-\frac{z}{2}}W_{k,\mu}(z)\,dz = \frac{\Gamma(\nu-\mu+\frac{1}{2})\Gamma(\nu+\mu+\frac{1}{2})}{\Gamma(\nu-k+1)}, \tag{4.9}$$

where $\mathrm{Re}(\mu\pm\nu) > -\frac{1}{2}$ and $W_{k,\mu}(z)$ is the Whittakar confluent hypergeometric function (see [30]).

Theorem 4.3. The following Whittakar transform representation for the extended Mittag-Leffler function in Equation (2.1) holds true:

$$\int_0^{\infty} z^{\nu-1}e^{-\frac{z}{2}}W_{\tau,\mu}(z)E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^{\rho};p)\,dz$$

$$= \frac{\Gamma(c)}{\Gamma(a)\Gamma(\gamma)} {}_4\psi_3^{\left((\mathcal{K}_l)_{l\in\mathbb{N}_0}\right)} \left[\begin{array}{c} (a,1),\ (\nu+\mu+\frac{1}{2},\rho), (\nu-\mu+\frac{1}{2}), \rho),\ \ (\gamma,1) \\ \\ (\beta,\alpha),\ (\nu-\tau+1,\rho),\ \ (c,1) \end{array} \middle| \ (z;p) \right]. \tag{4.10}$$

Proof. Using (4.9) and (2.1), it gives

$$\int_0^\infty z^{\nu-1} e^{-\frac{z}{2}} W_{k,\mu}(z) E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(xz^\rho;p)\,dz$$
$$= \sum_{n=0}^{\infty} \frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{x^n}{n!} \int_0^\infty z^{\nu+\rho n-1} e^{-\frac{z}{2}} W_{k,\mu}(z)\,dz$$
$$= \frac{\Gamma(c)}{\Gamma(a)\Gamma(\gamma)} \frac{1}{\Gamma(c-\gamma)} \sum_{n=0}^{\infty} B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0}}(\gamma+n,c-\gamma) \frac{\Gamma(a+n)}{\Gamma(\alpha n+\beta)} \frac{x^n}{n!}$$
$$\times \frac{\Gamma(\nu-\mu+\frac{1}{2}+\rho n)\Gamma(\nu+\mu+\frac{1}{2}+\rho n)}{\Gamma(\nu-\tau+1+\rho n)}.$$

In accordance with the definition of (2.8), we obatain the desired result (4.10). □

5. Fractional Calculus Approach

In this section, we derive certain interesting properties of the extended Mittag-Leffler function $E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)$ in Equation (2.1) associated with right-sided Riemann-Liouville fractional integral operator I_{b+}^{μ} and the right-sided Riemann-Liouville fractional derivative operator $D_{b^+}^{\mu}$, which are defined as (see, e.g., [11,22]):

$$\left(I_{b+}^{\mu}\varphi\right)(x) = \frac{1}{\Gamma(\mu)} \int_b^x (x-t)^{\mu-1}\varphi(t)\,dt \ \ (\mu\in\mathbb{C},\ \mathrm{Re}(\mu)>0), \tag{5.1}$$

and

$$\left(D_{b+}^{\mu}\varphi\right)(x) = \left(\frac{d}{dx}\right)^n \left(I_{b+}^{n-\mu}\varphi\right)(x) \ \ (\mu\in\mathbb{C},\ \mathrm{Re}(\mu)>0; n=[\mathrm{Re}(\mu)]+1), \tag{5.2}$$

where [x] means the greatest integer not exceeding real x.

Theorem 5.1. Let $b \in \mathbb{R}_+ = [0,\infty); \alpha,\beta,\gamma,\omega,a,c \in \mathbb{C};\ \mathrm{Re}\{\alpha,\beta,a,c\} > 0; \mu \in (0,1), \nu \in [0,1]$. Then, for $x > b$, the relation holds:

$$\left(I_{b+}^{\mu}\left[(t-b)^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^\alpha;p)\right]\right)(x)$$
$$= (x-b)^{\beta+\mu-1} E_{\alpha,\beta+\mu;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^\alpha;p), \tag{5.3}$$

$$\left(D_{b+}^{\mu}\left[(t-b)^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^\alpha;p)\right]\right)(x)$$

$$= (x-b)^{\beta-\mu-1} E_{\alpha,\beta-\mu;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^{\alpha};p). \tag{5.4}$$

Proof. By virtue of the formulas (Equations (5.1) and (2.1)), the term-by-term fractional integration and the application of the relation [31]:

$$(I_{b+}^{\mu}[(t-b)^{\beta-1}])(x) = \frac{\Gamma(\beta)}{\Gamma(\mu+\beta)}(x-b)^{\mu+\beta-1}, \ (\mu,\beta\in\mathbb{C}, \mathrm{Re}(\mu)>0, \mathrm{Re}(\beta)>0), \tag{5.5}$$

yield for $x > b$:

$$\begin{aligned}
&\left(I_{b+}^{\mu}\left[(t-b)^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^{\alpha};p)\right]\right)(x)\\
&= \left(I_{b+}^{\mu}\left[\sum_{n=0}^{\infty} \frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n, c-\gamma)}{B(\gamma,c-\gamma)} \frac{(a)_n}{\Gamma(\alpha n+\beta)} \frac{\omega^n (t-b)^{\alpha n+\beta-1}}{n!}\right]\right)(x)\\
&= (x-b)^{\beta+\mu-1} E_{\alpha,\beta+\mu;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^{\alpha};p).
\end{aligned} \tag{5.6}$$

Next, by Equations (5.2) and (2.1), we find that:

$$\begin{aligned}
&\left(D_{b+}^{\mu}\left[(t-b)^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^{\alpha};p)\right]\right)(x)\\
&= \left(\frac{d}{dx}\right)^{n} \left(I_{b+}^{n-\mu}\left[(t-b)^{\beta-1} E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^{\alpha};p)\right]\right)(x)\\
&= \left(\frac{d}{dx}\right)^{n} \left[(x-b)^{\beta+n-\mu-1} E_{\alpha,\beta+n-\mu;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\omega(t-b)^{\alpha};p)\right].
\end{aligned} \tag{5.7}$$

Applying Equation (3.17), we are led to the desired result Equation (5.4). □

6. Observations and Conclusion

The newly introduced function in (2.1) may find potential applications in various scientific fields. In physics, it can model anomalous relaxation and diffusion governed by fractional dynamics. In engineering, it provides flexible tools for describing viscoelastic behavior and systems with memory effects. In biology, such generalized Mittag-Leffler-type functions may represent complex growth or decay processes. These prospects emphasize the broader applicability of the proposed generalization.

Another important application of the Mittag-Leffler function is in fractional kinetic equations. These equations play a central role in modeling processes where memory effects and hereditary properties are significant. Unlike classical kinetic models, which assume exponential relaxation, fractional counterparts capture anomalous diffusion, non-exponential decay, and complex dynamical behavior. Such models have found applications in diverse fields, including physics, chemistry, biology, astrophysics, and engineering, where they describe phenomena such as reaction-diffusion systems, population growth, transport in disordered media, and relaxation dynamics in complex systems. As an illustration, in this section, we investigate solutions to a family of generalized fractional kinetic equations, extending the classical model of Saxena and Kalla (see [32]). The standard form of such an equation is given by

$$\mathcal{K}(\tau) - \mathcal{K}_0\varphi(\tau) = -\rho^{\nu}\, {}_0D_{\tau}^{-\nu}\mathcal{K}(\tau), \qquad (\Re(\nu)>0), \tag{6.1}$$

where $\mathcal{K}(\tau)$ denotes the number density of a species at time τ, $\mathcal{K}_0 = \mathcal{K}(0)$ is its initial density at $\tau = 0$, and ρ is a constant. Here, ${}_0D_\tau^{-\nu}$ represents the Riemann-Liouville fractional integral operator, defined for $\Re(\nu) > 0$ as (see [31, 33])

$${}_0D_\tau^{-\nu}\varphi(\tau) = \frac{1}{\Gamma(\nu)}\int_0^\tau (\tau - s)^{\nu-1}\varphi(s)\,ds. \tag{6.2}$$

The solution structure of such equations often involves the Mittag-Leffler function with two parameters, defined by

$$E_{\nu,\mu}(z) = \sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\nu n + \mu)}. \tag{6.3}$$

In what follows, we demonstrate that the generalized Mittag-Leffler function $E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(z;p)$ naturally arises in the solution of a generalized kinetic model.

Theorem 6.1. *The following result provides a model for a fractional kinetic equation. If $\omega > 0$ and $\nu > 0$, then the solution of the fractional kinetic equation*

$$\mathcal{K}(\tau) - \mathcal{K}_0\,\tau^{\beta-1}E_{\alpha,\beta;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\tau^\alpha;p) = -\omega^\alpha\,{}_0D_\tau^{-\alpha}\mathcal{K}(\tau), \tag{6.4}$$

is given explicitly by

$$\mathcal{K}(\tau) = \mathcal{K}_0\,\tau^{\beta-1}\sum_{r=0}^{\infty}(\omega\tau)^{r\alpha}\,E_{\alpha,\beta+\alpha r;a}^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0};\gamma,c)}(\tau^\alpha;p). \tag{6.5}$$

Proof. Taking Laplace transforms and using the identity (see [33])

$$\mathcal{L}\left[{}_0D_\tau^{-\nu}f(\tau);s\right] = s^{-\nu}F(s), \qquad F(s) = \int_0^\infty e^{-s\tau}f(\tau)\,d\tau, \quad \Re(s) > 0,$$

we obtain from (6.4)

$$\mathcal{K}(s) - \mathcal{K}_0\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\})}(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)}\frac{(a)_n}{n!}\,s^{-(\alpha n+\beta)} = -\omega^\alpha s^{-\alpha}\mathcal{K}(s).$$

Hence

$$\mathcal{K}(s)\left(1 + \omega^\alpha s^{-\alpha}\right) = \mathcal{K}_0\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\})}(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)}\frac{(a)_n}{n!}\,s^{-(\alpha n+\beta)}.$$

For $|\omega^\alpha s^{-\alpha}| < 1$ (for instance $|s| > |\omega|$) we may expand

$$\left(1 + \omega^\alpha s^{-\alpha}\right)^{-1} = \sum_{r=0}^{\infty}(-1)^r\omega^{\alpha r}s^{-\alpha r},$$

and therefore

$$\mathcal{K}(s) = \mathcal{K}_0\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\})}(\gamma+n, c-\gamma)}{B(\gamma, c-\gamma)}\frac{(a)_n}{n!}\sum_{r=0}^{\infty}(-1)^r\omega^{\alpha r}s^{-(\alpha r+\alpha n+\beta)}.$$

The interchange of the two sums and the subsequent Laplace inversion are justified for sufficiently large $\Re(s)$ by absolute/uniform convergence under the standing hypotheses on α and the growth of the extended Beta term. Using

$$\mathcal{L}^{-1}[s^{-\nu};\tau]=\frac{\tau^{\nu-1}}{\Gamma(\nu)},\qquad \Re(\nu)>0,$$

we obtain

$$\mathcal{K}(\tau)=\mathcal{K}_0\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{n!}\sum_{r=0}^{\infty}\frac{(-1)^r\omega^{\alpha r}\,\tau^{\alpha r+\alpha n+\beta-1}}{\Gamma(\alpha r+\alpha n+\beta)}.$$

Comparing with the definition of the generalized Mittag-Leffler function (6.3) yields the asserted formula

$$\mathcal{K}(\tau)=\mathcal{K}_0\,\tau^{\beta-1}\sum_{r=0}^{\infty}(-1)^r(\omega\tau)^{r\alpha}\,E_{\alpha,\beta+\alpha r;a}^{(\{\mathcal{K}_l\};\gamma,c)}(\tau^{\alpha};p)$$

and the proof is complete. □

Remark 6.1. According to the definition (6.3), the solution of the fractional kinetic equation (6.4) can alternatively be expressed as

$$\begin{aligned}\mathcal{K}(\tau)&=\mathcal{K}_0\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n}{n!}\sum_{r=0}^{\infty}\frac{(-1)^r\omega^{\alpha r}\,\tau^{\alpha(r+n)+\beta-1}}{\Gamma(\alpha(r+n)+\beta)}\\&=\mathcal{K}_0\,\tau^{\beta-1}\sum_{n=0}^{\infty}\frac{B_p^{(\{\mathcal{K}_l\}_{l\in\mathbb{N}_0})}(\gamma+n,c-\gamma)}{B(\gamma,c-\gamma)}\frac{(a)_n\,\tau^{\alpha n}}{n!}\,E_{\alpha,\alpha n+\beta}(-(\omega\tau)^{\alpha}).\end{aligned}\tag{6.8}$$

This alternative form follows directly from the series definition of the two-parameter Mittag-Leffler function (1.13). Future investigations may focus on extending the present results in several directions. One promising line of research is to explore the solutions of fractional differential equations defined via operators such as the Caputo-Katugampola derivative in terms of the newly defined generalized Mittag-Leffler function [16]. Further studies could also address numerical methods, asymptotic behavior, and potential physical interpretations associated with these generalized models.

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